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Influence of the bed mass on its fluidization characteristics

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Abstract

Two series of fluidization tests were carried out on two test models with catalyst, alumina and sand particles to determine the bed mass influence on the characteristics at the minimum of fluidization. The main conclusions of the present study are: (i) the measured minimum fluidization velocities increased with the inventory whatever were the solid and the test rig used; (ii) the measured bed porosity at minimum fluidization decreased with the increase of the bed inventory; (iii) the definition of the minimum fluidization velocity by the balance between weight and drag forces and some usual mathematical modeling attempts were not able to describe the minimum fluidization increase with the bed inventory; (iv) the addition of a complementary consolidation effect in the force balance was able to match the obtained experimental results; (v) the consolidation effect sorted out the solids in the same order than the one given by their flowability as measured in shearing test.

Keywords: Fluidization; Minimum fluidization velocity; Bed porosity; Consolidation; Inventory

1. Introduction

Various applications of fluidized beds in process engineering impose to consider the mass of the fluidized bed bore by the fluidizing gas or liquid, as a major operating parameter. As a matter of fact, the necessary improvement of the process productivity often demand to increase the inventory mass in the reactor. For instance in the case of environmental applications of fluidized beds such as water or gas depollution or soil remediation, the increase of the bed inventory may improve the global depollution efficiency of the operation provided that the auxiliaries may endure the complementary flows or pressure drops. The Environmental Process Engineering Laboratory of the Ecole des Mines de Nantes currently develops gas–solid contactors operating at fixed or fluidized bed regime. A great part of these processes are devoted to mass transfer between gas and solids such as the separation with or without the destruction of odorous or volatile organic compounds (VOCs) by physical, chemical or biological routes [1]. The increase of regulations strictness for air quality control have lead to undertake several programs to improve the efficiency of the processes that have been previously developed [2]. Such improvements can be obtained either by the design of the processes themselves, either by the change of their operating conditions. For instance a VOC separation process by condensation at relatively low temperatures on porous particles in circulating fluidized bed was developed in the above mentioned laboratory before patenting it. In the particular case of this process, on the one hand, the fluidized bed's inventory can be increased at will to improve the pollutants abatement whilst, on the other hand, the inventory weight also increases due to the liquid fraction captured by the porous particle void volume and thus to their density increase [3]. It can thus be seen that in this particular application, the design and the operation of the fluidized bed requires to keep a good quality of fluidization as well as a sufficient long residence time for the gas (a low gas velocity). The designer and the operator have thus to control the gas velocity slightly above the minimum fluidization velocity with sufficient accuracy and certainty. The design of fluidized beds devoted to gas biofiltration is another example of application that also demands an optimization of the fluidized bed inventory and the gas superficial velocity. As a matter of fact, such a bioreactor works with a rather high solid inventory and a low gas velocity because bioreactions of VOC destruction demand a great contact time between micro-organisms and the gas pollutants as well as a good fluidization to allow the maximization of the solid surface offered to the gas. These two preceding applications of fluidized beds emphasize the relevance of studying the evolution of the minimum fluidization as a function of the bed inventory. This paper aims at demonstrating and interpreting the effect of the inventory on the bed characteristics at minimum fluidization: its minimum fluidizing velocity and its bed voidage. Experiments

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Nomenclature

a	first coefficient of Ergun's equation ($kg/m^3\,s$)
b	second coefficient of Ergun's equation (m^{-1})
d_p	particle specific surface mean diameter (m)
g	gravity acceleration (m/s^2)
G	gas mass-flow rate per unit of bed cross-section ($=\rho U$) ($kg/m^2\,s$)
H_{mf}	bed height at minimum fluidization (m)
H_0	atmospheric pressure in gas height at bed surface conditions (m)
K	pseudo-consolidation coefficient (–)
P	gas pressure (Pa)
$\mathcal{P}$	piezometric gas pressure ($=P + g\rho z$) (Pa)
$P(H_{mf})$	gas pressure at bed free surface (Pa)
$P(0)$	gas pressure at bed bottom (Pa)
S	riser cross-section (m^2)
U	superficial gas velocity (m/s)
U_{mf}	minimum fluidization gas velocity (m/s)
W	bed weight ($kg\,m/s^2$)
W_p	apparent weight of solids per unit of bed cross-section ($kg\,m/s^2$)
z	height above the reference (m)

Greek symbols

Δ	difference between bottom and free surface of the solid layer (–)
ε_{mf}	bed voidage at minimum fluidization (–)
μ	gas dynamic viscosity (Pa s)
ρ	gas density (kg/m^3)
ρ_p	solid apparent density (kg/m^3)
$\rho(H_{mf})$	gas density at bed free surface (kg/m^3)

were carried out with three solids on two different test rigs where fluidization velocity and bed porosity were measured. These experiments were then interpreted by the mean of different physics models relying upon the interaction between gas and particles and the particle–particle interaction.

The effect of bed inventory on fluidized bed characteristic is seldom studied. Cranfield and Geldart [4] have measured the minimum fluidization velocity for alumina beads of 1520 μm diameter and different bed inventory in a 2D model. The minimum velocity regularly increased from 0.51 to 0.64 m/s when the bed height varied from 0.05 to 0.3 m. However, other experiments carried out by the same authors on a 3D model with nearly similar particles do not confirm this trend. Denloye [5] mentioned that the minimum fluidization velocity of a 1020 μm sand increases from 0.4 to 0.45 (11%) when the static bed height increases from 5 to 30 cm. The author explained that it is probably due to the increase from 94 to 99% of the ratio of bed pressure drop to weight of the particles per unit area of bed cross-section. Thonglimp et al. [6] have studied the influence of particle diameter and density, the bed inventory and the column diameter on the minimum fluidization velocity and the bed expansion. The experiments carried out by these authors with glass beads from B and D Geldart classes cover a relatively broad range of the parameters: particle diameter from 112.5 to 2125 μm; column diameters of 5, 9.5, 19.4 and 43.4 cm; bed inventory varying from 50 to 500 kg/m^2. Their results show that the measured minimum fluidization of glass beads of class D can be considered as increasing with the bed inventory if it is assumed that the velocity has been given by the authors on the basis of a nearly constant gas pressure at the free surface. For class B particles used by Thonglimp et al., the minimum fluidization velocity remains nearly constant. In a more recent paper, Tannous et al. [7] have given complementary results of experiments carried out on the same rigs as Thonglimp et al. with glass beads of class D. Tannous et al. have used three methods to measure the minimum fluidization velocity and have confirmed the preceding conclusions concerning class D solids. Other studies demonstrated that the inventory increase implies an increase of the minimum fluidization velocity mainly because the gas expansion phenomenon delays the fluidization of the bottom whereas the upper part is already fluidized [8–10].

2. Materials and methods

Three types of solids have been fluidized in two different cold models by varying the inventory present in the test rigs. The first of the two cold models consisted in a 0.192 m diameter of a Perspex column equipped with a porous plate of 40% opening area, whilst the second one was made out of steel and had a square section of 0.096 m side and a grid made of five tuyeres having four nozzles for air injection each.

The pressure drop of the bed was measured with the help of precise sensors whilst the gas mass-flow rate was measured thanks to two mass-flow rate sensors having a range of, respectively, 0–100 and 0–5 N l/h. They were used according the range of flow rate necessary to obtain the bed pressure drop diagram versus the gas flow rate for minimum fluidization determination. In contrast with the steel model, the Perspex model allowed to measure the bed height at fluidization onset by measurement of the bed height.

The tested solids were: (1) alumina, (2) river sand and (3) spent cracking catalyst and have covered the Geldart powder categories A and B. Their respective mean harmonic diameter was 89, 183 and 77 μm as measured by a Laser Coulter; their apparent density, i.e. particle density, was, respectively, 2000, 2640 and 1550 kg/m^3.

Fig. 1 shows examples of fluidization (increasing air flow rate) and defluidization (decreasing air flow rate) curves drawn for sand for the minimum and the tested weight, 5 and 20 kg on the Perspex cold model. The fluidization curves are

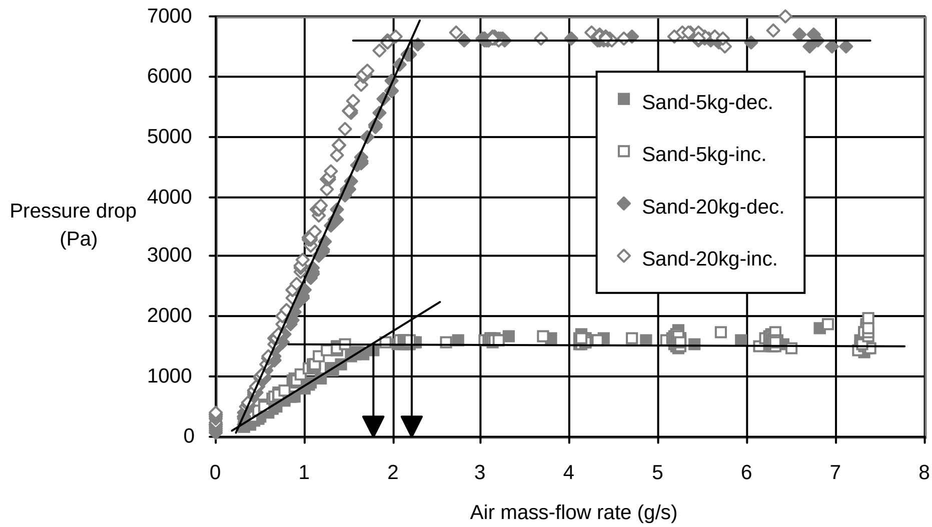

Fig. 1. Fluidization and defluidization curves for bed sand of 5 and 20 kg in the Perspex cold model equipped with a porous plate.

a little on the left of the defluidization ones and yield minimum fluidization flow rates smaller than those obtained by defluidization. The minimum fluidization flow rate or velocity mentioned in this study are the ones obtained with the defluidization curves.

3. Results

Fig. 2 shows the minimum fluidization velocity obtained by the defluidization curves for the three solids and the two models. The velocity mentioned in Fig. 2 is the gas velocity calculated after the gas mass-flow rate accounting for the pressure and temperature conditions at the free surface of the bed. Air temperature is thus bed temperature and air pressure is the atmospheric pressure. For the three solids, there is an increase of the minimum of fluidization with the bed inventory. It can also be seen in Fig. 2 that this increase appears both for the Perspex and the steel models. Moreover, the minimum fluidization is nearly the same whatever the test rig used for its determination.

The total bed pressure drop when the bed was at its minimum fluidization was also compared to the weight per unit area. Fig. 3 gives the so-called channeling coefficient which is the difference between the bed pressure drop at minimum fluidization and the bed weight per unit of bed cross-section effectively poured in the rigs reported to the bed weight per unit of bed cross-section. A high channeling coefficient corresponds to a high fraction of the bed weight not contributing to the gas pressure drop. As far as the Perspex column is concerned, the deviation remained relatively low for the sand and decreased as the weight increased. In contrast alumina and catalyst exhibited an increasing channeling coefficient with weight, the catalyst's one being more significant.

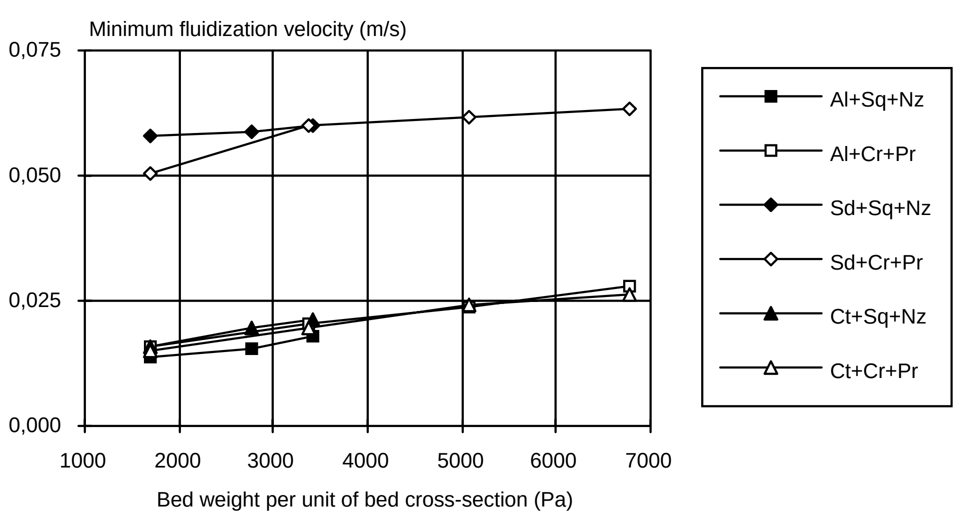

Fig. 2. Minimum fluidization for various bed inventories obtained for alumina (Al); sand (Sd); catalyst (Ct) with the steel square model (Sq) and nozzle grid (Nz) and the Perspex cylindrical model (Cr) and the porous plate (Pr).

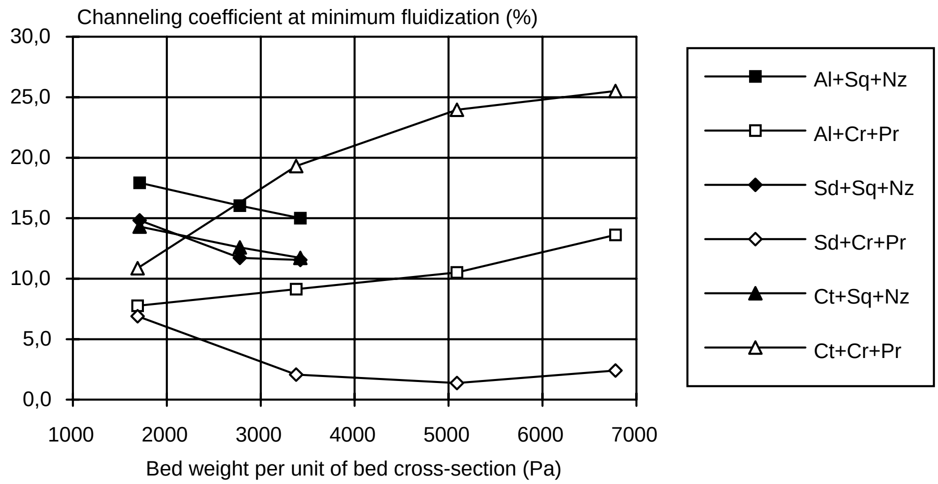

Fig. 3. Channeling coefficient in percentage giving the relative deviation of pressure drop compared to bed weight per unit of bed cross-section at minimum fluidization for various bed inventories given in Pa; alumina (Al); sand (Sd); catalyst (Ct) with the steel square section model (Sq) and the nozzle grid (Nz) or the Perspex cylindrical model (Cr) and the porous plate (Pr).

The steel test rig showed a different behavior: the channeling coefficient decreased when bed weight increased for the three solids. The design of the air distributor of the steel test rig explains one part of the channeling. As a matter of fact, the air injection is located at 1.5 cm above the bottom of the vessel and because the air injection is horizontal, a part of the particles are not bore by the gas flow. The volume of solids is nearly equal to 10^{-4} cm^3 and depending on the solids whose bulk density can vary from 940 kg/m^3 for the catalyst, 1200 kg/m^3 for alumina to 1540 kg/m^3 for the sand, this volume may represent nearly 6–10% of 1.6 kg and obviously less for bigger inventories.

As said before, the bed height was measured during the tests in Perspex cold model. The bed voidage have then been calculated knowing the apparent density of particles. Fig. 4 shows that bed voidage at minimum fluidization decreased with the increase of bed inventory whatever the solid used is. A slight overall settling of the bed with a decrease of its voidage at the minimum fluidization velocity appeared when its inventory was increased, notwithstanding the fluidization was studied by decreasing gas flow rate. However, it must be kept in mind that the bed height measurement accuracy is poor and that the bed height was measured from time to time but was not recorded at exactly the minimum fluidization since it is unknown before the end of the whole series of measures.

4. Discussion

The objective of these tests was to show the influence of the bed inventory on its characteristics at minimum fluidization such as its minimum fluidization velocity or its bed voidage. In other words, the objective was thus to evaluate the behavior of one layer isolated by thought in a fluidized bed and how it is influenced by the surrounding

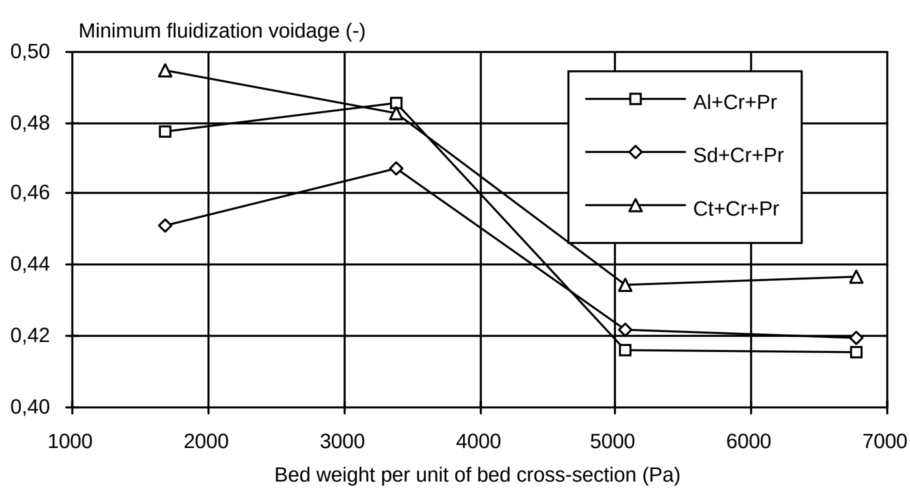

Fig. 4. Bed voidage at minimum fluidization velocity as a function of the bed weight per unit of bed cross-section; alumina (Al); sand (Sd); catalyst (Ct) with the Perspex cylindrical model (Cr) and the porous plate (Pr).

layers modifying its ability to become fluidized or to aerate itself.

The results of Fig. 2 were first interpreted thanks to the minimum fluidization balance which states that at minimum fluidization the pressure drop of the gas is equal to the weight of solids per unit of bed cross-section. This equilibrium was written assuming, firstly that the pressure drop can be described by Ergun's law [11] and, secondly by the law proposed by Delebarre [9] where the progressive expansion of the gas is accounted for. Notice that these two approaches assume a uniform porosity throughout the bed that might influence the pressure drop evaluation. The two balances were written with a uniform bed voidage equal to the one measured at the minimum fluidization during the experiments using the following equations and eliminating the piezometric pressure gradient with the equation giving the specific weight. Firstly Ergun's law (Eq. (1)) is used with the fluidization definition (Eq. (4)) which states that the piezometric pressure difference is equal to the apparent weight of solids by unit of bed cross-section [12]. The piezometric pressure $\mathcal{P}$ drop given by

$$\frac{\Delta\mathcal{P}}{H_{\mathrm{mf}}} = aU_{\mathrm{mf}} + b\rho U_{\mathrm{mf}}^2 \tag{1}$$

with

$$a = 150\frac{(1-\varepsilon_{\mathrm{mf}})^2}{\varepsilon_{\mathrm{mf}}^3}\frac{\mu}{d_{\mathrm{p}}^2} \tag{2}$$

and

$$b = 1.75\frac{1-\varepsilon_{\mathrm{mf}}}{\varepsilon_{\mathrm{mf}}^3 d_{\mathrm{p}}} \tag{3}$$

is thus combined with the minimum fluidization definition

$$\Delta\mathcal{P} = W_{\mathrm{p}} = g(1-\varepsilon_{\mathrm{mf}})(\rho_{\mathrm{p}}-\rho)H_{\mathrm{mf}} \tag{4}$$

where the gas density is often neglected compared to the particle density. The system of equations can be solved either to find U_{mf} knowing the other variables and particularly $\varepsilon_{\mathrm{mf}}$, either to calculate $\varepsilon_{\mathrm{mf}}$ knowing the minimum fluidization velocity U_{mf}. In the first case, the system reduces itself to a second degree equation in U_{mf}; in the other case the equation with $\varepsilon_{\mathrm{mf}}$ as an unknown is a third degree equation that can be solved explicitly with the Cardan's formula [13]. Notice that solving the system of equations (1)–(4) do not use at any time any hypothesis on a relationship between some characteristics of the particles such as their sphericity and other operating characteristics during fluidization such as their bed voidage. As a result, the minimum calculated fluidization U_{mf} decreases when the measured bed voidage $\varepsilon_{\mathrm{mf}}$ decreased. And conversely, the calculated bed voidage increases when the measured minimum fluidization velocity increased.

A generalization of Ergun's law was recently proposed to account for the gas expansion when it flows through the porous medium. In this case, the pressure drop is written as follows:

$$P(0) = P(H_{\mathrm{mf}})\left\{\frac{H_{\mathrm{mf}}}{2H_0} + \left[\left(\frac{H_{\mathrm{mf}}}{2H_0}+1\right)^2 + 2\frac{(a+bG)G}{\rho(H_{\mathrm{mf}})^2 g}\frac{H_{\mathrm{mf}}}{H_0}\right]^{1/2}\right\} \tag{5}$$

where H_0 denotes the atmospheric pressure at free bed surface and G the gas mass-flow rate per unit of bed cross-section, id est the product of gas velocity by gas density. Similarly, the pressure can be eliminated between Eqs. (1) and (5) and the minimum fluidization is thus a function of several parameters whose bed voidage. And similarly, the same conclusion stands: U_{mf} decreases when the bed voidage $\varepsilon_{\mathrm{mf}}$ decreases.

Fig. 5 illustrates the results of the comparison of calculated minimum fluidization velocity using Eqs. (1) and (4) and the measured bed voidage at minimum fluidization.

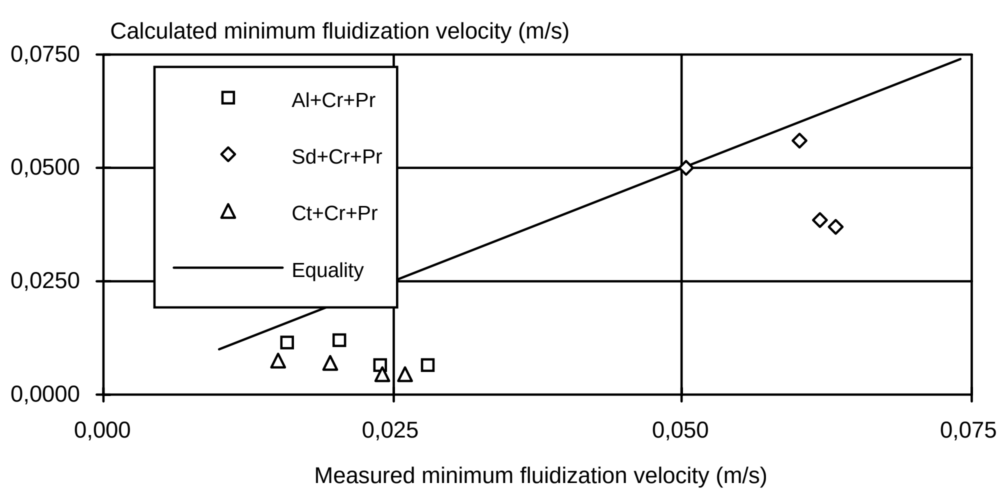

Fig. 5. Comparison of minimum fluidization velocity as measured and calculated by the equality between the gas pressure drop calculated with Ergun's equation and the bed weight per unit area. Bed voidages used in these calculations were the measured ones. Alumina (Al); sand (Sd); catalyst (Ct) with the Perspex cylindrical model (Cr) and the porous plate (Pr).

The results did not differ much from the ones obtained with Eqs. (1) and (5). The calculated values are lower than the measures and furthermore the calculations were unable to represent the empirical minimum fluidization velocity increase with the bed inventory. As a matter of fact, the bed voidage effect is predominant in these calculations and this parameter was shown to decrease with bed inventory in contrast with the minimum fluidization velocity that was empirically observed as increasing. Conversely, the models of pressure drop used for these calculations state that the smaller the bed voidage, the smaller the minimum fluidization velocity.

Other attempts of calculations were carried out using the same elimination of variables but assuming that only a part of the bed was bore by the gas flow. This channeling effect is equivalent to use the following equation (Eq. (6)) instead of Eq. (4) with a coefficient K:

$$\Delta\mathcal{P} = Kg(1-\varepsilon_{\rm mf})(\rho_{\rm p}-\rho)H_{\rm mf} \tag{6}$$

where K was in this case the coefficient of the effective fluidized weight which is defined by Eq. (7). The difference of K to 1 or 100% is the channeling coefficient that has been given in Fig. 3:

$$K = \frac{\Delta P}{W/S} \tag{7}$$

The weight effectively fluidized accounting for the channeling has the following effect on the calculated minimum fluidization: the minimum fluidization velocity decreases when the coefficient K decreases. In particular, if the coefficient K becomes smaller than the ideal case with coefficient K equal to unity which corresponds to the total bed weight being fluidized without channeling nor dead zones, the minimum fluidization velocity decreases. Thus the increase of the measured minimum fluidization velocities may be attributed to the disappearance of channeling (increase of K). Fig. 3 thus indicates that this explanation only stands for the case of the sand where the channeling decreased with the increase of the inventory. Nevertheless, the calculations with the two sets of equations proved that this increase was not sufficient to counterbalance the effect of the bed voidage and to fit the measured values.

A lot of works on the fluidization of cohesive beds introduced an interparticle force that is expressed as a multiple of the buoyant weight [14]. Similarly, it was assumed that the inventory increase of the present study experiments caused a slight compaction or settling (decrease of bed voidage) and simultaneously a consolidation of the bed that was reducing particle free space around themselves and increasing contacting forces. The minimum fluidization velocity by defluidization as used in these tests, id est the onset of defluidization, may thus have happened at larger gas velocity. The inventory increase provoked a settling of the bed as shown by the bed voidage measurement that prevented the particles to fluidize freely. Thus the defluidization appeared at larger fluidization decreasing gas velocities.

Fig. 6 gives the calculations of the coefficient K renamed as the pseudo-consolidation coefficient which would be necessary to have the minimum fluidization calculated by Eqs. (1)–(6) equal to the measurements. It can be seen that for all solids, the K-values increased nearly linearly with the bed inventory showing an increased consolidation effect. It is also interesting to notice that the Geldart-A solids such as alumina and catalyst give a high value of K compared to the Geldart-B sand. This effect should not be confused with desaeration kinetics of powder where fine solids are known to desaerate much slower than sand-like solids. The defluidization curves of these measures were obtained with duration much greater than the desaeration rate of the finer solids. Also notice that the values exhibited by these calculations are much lower than the value proposed by authors in the case of cohesive powders: Rhodes et al. have quoted values of K in the range of 20–50 for fluidization of particles with interparticle forces [14]. Another remarkable feature of Fig. 6 is the trends of the three sets of experimental values that exhibit intercepts of the strait lines at small inventories which give K-values near 1. Well then coefficient K equal to unity corresponds to the hypothesis of

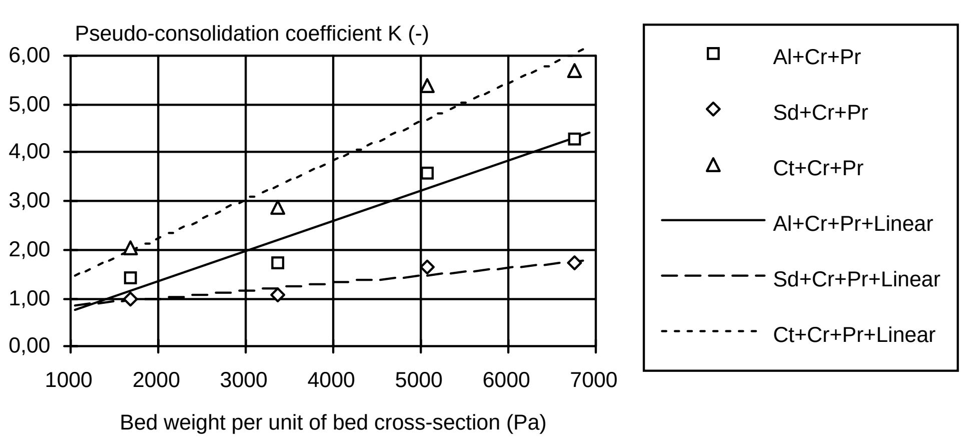

Fig. 6. Variation of the so-called packing coefficient K as a function of the bed inventory. Alumina (Al); sand (Sd); catalyst (Ct) with the Perspex cylindrical model (Cr) and the porous plate (Pr).

a fair prediction of the minimum fluidization velocity with pressure drop as given by Ergun's equation and equal to the weight of the bed per unit of bed cross-section area. It has thus to be concluded that the definition and the prediction of the minimum fluidization velocity thanks to an equality between the pressure drop equation such as Ergun's one and the weight of solids by unit of bed cross-section area are well adapted and improved for bed inventory rather small.

Moreover, the slopes of the three straight lines of Fig. 6 are in the same order than the three flowability indices of the three solids determined by their flow function slopes after shearing tests, the sand being the more flowable (its flow function is practically horizontal), the catalyst the less. The bed inventory effect described by the pseudo-consolidation coefficient K evolution looks somehow similar to the powder flow ability as a function of the applied consolidation [15,16]. The coefficient K as a function of the inventory is a sort of resistance to fluidization due to consolidation and has a similar role than the reciprocal of the slope of the unconfined yield stress versus the steady state major principal stress. If the inventory is increased, then the fluidization will occur at higher gas velocity as well as the failure of powders given by shearing tests. Moreover, Barletta et al. [17] recently presented shearing test results in an aerated shearing cell: this cell allowed to aerate the powder (below its fluidization) during its preconsolidation and its shearing. The main conclusion of the authors is that, if the force balance and the stress determination accounts for the drag forces and the pressure forces induced by the gas flow through the powder, then the flow function of powder does not depend much on the aeration rate. The main effect of aeration is to decrease the local normal solid stress not to change the intrinsic properties of the powder: its flow index is not affected by aeration. In the case of the present (de)fluidization tests, the inventory consolidates the fluidized bed so that the gas velocity necessary to avoid the complete settling of the bed has to counterbalance the lower mobility of particles induced by the decreasing porosity. The intrinsic fluidizability of powders can be found at low values of inventories and low values of K. In these conditions, the definition of fluidization as a balance between the gas pressure drop and the bed weight per unit of bed cross-section stands. However, the bed inventory increase induces a sort of consolidation of particles as well as a change in its bed characteristic that the gas flow has to compensate to maintain the fluidization.

5. Conclusions

Two series of tests were carried out on two test models with three types of solids in order to determine the bed mass influence on its characteristics at the minimum of fluidization. Different models were then attempted to explain the experimental results. The main conclusions of the present study are:

- the measured minimum fluidization velocities increased with the inventory whatever were the solid and the test rig used;
- the measured bed porosity at minimum fluidization decreased with the increase of the bed inventory;
- the channeling coefficient has different trends according to the solid types and probably the air distributor;
- the definition of the minimum fluidization velocity by the balance between weight and drag forces and some usual mathematical modeling attempts were not able to describe the minimum fluidization increase with the bed inventory;
- the addition of a complementary consolidation effect in the force balance was able to match the obtained experimental results;
- the consolidation effect sorted out the solids in the same order than the one given by their flowability as measured in shearing test.

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References

[1] P. Le Cloirec, Les composés organiques volatils dans l'environnement, Tech et Doc, Paris, 1998.
[2] C. Faur-Brasquet, H. Métivier-Pignon, P. Le Cloirec, Activated carbon cloths in water and wastewater treatments, Res. Adv. Water Res. 2 (2002).
[3] W. Yazbek, A. Delebarre, Application du froid à la séparation de polluants par condensation en lit fluidisé, Revue Générale du Froid N° 1034 (2003) 15.
[4] R.R. Cranfield, D. Geldart, Large particles fluidization, Chem. Eng. Sci. 29 (1974) 935.
[5] A.O. Denloye, Bed expansion in a fluidized bed of large particles, J. Powder Bulk Technol. 6 (3) (1982) 11.
[6] V. Thonglimp, N. Hiquily, C. Laguérie, Vitesse minimale de fluidisation et expansion des couches fluidisées par un gaz, Powder Technol. 38 (1984) 233.
[7] K. Tannous, M. Hemati, C. Laguérie, Caractéristiques de fluidisation et expansion des couches fluidisées de particules de la catégorie D de Geldart, Powder Technol. 80 (1994) 55.
[8] K. Kusakabe, T. Kuriyama, S. Morooka, Fluidization of fine particles at reduced pressure, Powder Technol. 58 (1989) 201.
[9] A. Delebarre, Does the minimum fluidization exist, J. Fluids Eng. 124 (2002) 595.
[10] A. Delebarre, E. Levol, J.-M. Morales, Does the minimum fluidization characterize a powder? in: Proceedings of the World Congress on Particle Technology, vol. 4, Paper No. 401, Sydney, Australia, July 21–25, 2002. ISBN 085 825 7947.
[11] S. Ergun, Fluid flow through packed columns, Chem. Eng. Progress 48 (2) (1952) 89.

[12] R.-H. Jean, L.-S. Fan, On the model equations of Gibilaro and Foscolo with corrected buoyancy force, Powder Technol. 72 (1992) 201.
[13] J. Lelong-Ferrand, J.-M. Arnaudiès, Cours de mathématiques, Algèbre, vol. 172, Dunod, Paris, 1998.
[14] M.J. Rhodes, X.S. Wang, M. Nguyen, P. Stewart, K. Liffman, Onset of cohesive behaviour in gas fluidized beds: a numerical study using DEM simulation, Chem. Eng. Sci. 56 (2001) 4433.
[15] J. Schwedes, D. Schulze, Measurement of flow properties of bulk solid, Powder Technol. 61 (1990) 59.
[16] J. Schwedes, Consolidation and flow of cohesive bulk solids, Chem. Eng. Sci. 57 (2002) 287.
[17] D. Barletta, G. Ferrari, K. Johanson, M. Poletto, B. Scarlett, The role of aeration on the incipient failure of powders, in: Proceedings of the Sixth Conference on Fluid–Particle Interactions, Barga, August 25–30, 2002.